Chapter 19 – Further Applications of the LL scales

19.1 Reciprocals

The LL (e^x) scales and the corresponding LL_0 (e^{-x} or $\frac{1}{e^x}$)Scales are the reciprocal of each other.

Example 1: $\frac{1}{4.1}$ = 0.244

1. Set the hair line over 4.1 on the LL_3 scale.
2. Under the hair line read off 0.244 on the LL_{03} scale.

Example 2: $\frac{1}{0.9335}$ = 1.071

1. Set the hair line over 0.9344 on the LL_{01} scale.
2. Under the hair line read off 1.071 on the LL_1 scale as the answer.

Note

(a) The reciprocal of any number from 0.000045 to 22,000 (10^{-5} to 10^5 for Slide Rules with extended scales) can be obtained by locating the number on the particular e^x or e^{-x} scale and reading off on the corresponding e^{-x} or e^x scale as in examples above.

(b) Even for a Slide Rule with LL_0 and LL_{00} scales, there is a gap between 0.9991 and 1.0009, but this range would not often be encountered. For number between 0.1 and 6, the LL scales give more accuracy the CI and D scales, while outside this range (i.e. greater than 6 and less then 0.1) the CI and D scales are more accurate. One of the greatest advantages of the LL scales for reciprocals, is that the decimal point is read directly off the scales.

e.g. 270 on the LL_3 scale gives $\frac{1}{270} = 0.0037$ on the LL_{03} scale.

Exercise 19(a)

(i)	$\frac{1}{23}$ =	(v)	$\frac{1}{0.98}$ =
(ii)	$\frac{1}{3.9}$ =	(vi)	$\frac{1}{0.9}$ =
(iii)	$\frac{1}{1.46}$ =	(vii)	$\frac{1}{0.48}$ =
(iv)	$\frac{1}{1.051}$ =	(viii)	$\frac{1}{0.0032}$ =

19.2 Tenth and Hundredth Powers and Roots

Example: Find 2.5^{10}, $\sqrt[10]{2.5}$, $\frac{1}{2.5^{10}}$, and $\frac{1}{\sqrt[10]{2.5}}$. (Fig 19.2)

1. Set the hair line over 2.5 on the LL_2 scale.

Under the hair line red off –

2. 9,500 on the LL_3 scale as the value of 2.5^{10}
3. 1.096 on the LL_1 scale as the value for $\sqrt[10]{2.5}$.
4. 0.000105 on the LL_3 scale as the value for $\frac{1}{2.5^{10}}$.

5. 0.9124 on the LL_{01} scale as the value for $\frac{1}{\sqrt[10]{2.5}}$.

Further Examples:
For 'a' on the LL scale as shown, the other LL scales give –

LL_3	LL_2	LL_1	LL_0	LL_{00}	LL_{01}	LL_{02}	LL_{03}
a	$\sqrt[10]{a}$	$\sqrt[100]{a}$	$\sqrt[1000]{a}$	$\frac{1}{\sqrt[1000]{a}}$	$\frac{1}{\sqrt[100]{a}}$	$\frac{1}{\sqrt[10]{a}}$	$\frac{1}{a}$
a^{10}	a	$\sqrt[10]{a}$	$\sqrt[100]{a}$	$\frac{1}{\sqrt[100]{a}}$	$\frac{1}{\sqrt[10]{a}}$	$\frac{1}{a}$	$\frac{1}{a^{10}}$
a^{100}	a^{10}	a	$\sqrt[10]{a}$	$\frac{1}{\sqrt[10]{a}}$	$\frac{1}{a}$	$\frac{1}{a^{10}}$	$\frac{1}{a^{100}}$
a^{1000}	a^{100}	a^{10}	a	$\frac{1}{a}$	$\frac{1}{a^{10}}$	$\frac{1}{a^{100}}$	$\frac{1}{a^{1000}}$
$\frac{1}{a^{1000}}$	$\frac{1}{a^{100}}$	$\frac{1}{a^{10}}$	$\frac{1}{a}$	a	a^{10}	a^{100}	a^{1000}
$\frac{1}{a^{100}}$	$\frac{1}{a^{10}}$	$\frac{1}{a}$	$\frac{1}{\sqrt[10]{a}}$	$\sqrt[10]{a}$	a	a^{10}	a^{100}
$\frac{1}{a^{10}}$	$\frac{1}{a}$	$\frac{1}{\sqrt[10]{a}}$	$\frac{1}{\sqrt[100]{a}}$	$\sqrt[100]{a}$	$\sqrt[10]{a}$	a	a^{10}
$\frac{1}{a}$	$\frac{1}{\sqrt[10]{a}}$	$\frac{1}{\sqrt[100]{a}}$	$\frac{1}{\sqrt[1000]{a}}$	$\sqrt[1000]{a}$	$\sqrt[100]{a}$	$\sqrt[10]{a}$	a

Exercise 19(b)

(i) $1.08^{10} =$

(ii) $1.08^{100} =$

(iii) $\frac{1}{1.08^{10}} =$

(iv) $\frac{1}{1.08^{100}} =$

(v) $\sqrt[10]{54} =$

(vi) $\sqrt[100]{54} =$

(vii) $\frac{1}{\sqrt[10]{54}} =$

(viii) $\frac{1}{\sqrt[100]{54}} =$

(ix) $\sqrt{0.65} =$

(x) $0.65^{10} =$

(xi) $\frac{1}{\sqrt[10]{.65}} =$

(xii) $\frac{1}{0.65^{10}} =$

(xiii) $\sqrt[10]{9.5} =$

(xiv) $\sqrt[100]{.03} =$

(xv) $1.7^{10} =$

(xvi) $0.97^{100} =$

Note:

(a) There is no difficulty with locating decimal points, as all values are read off the particular LL scale.

(b) Recall $\sqrt[10]{a} = a^{\frac{1}{10}} = a^{0.1}$

$$\sqrt[100]{a} = a^{\frac{1}{100}} = a^{0.01}$$

$$\sqrt[1000]{a} = a^{\frac{1}{1000}} = a^{0.001}$$

19.3 Positive Numbers to Any Powers

The LL scales can be used to obtain a^N for a > 0 and N, any power (positive, negative or fractional). The procedure is as follows: For 'a' on the LL scale, the D scale give 'ln a' (see unit 18). If we multiply this value of 'ln a' on the

D scale by 'N' (using the C and D scale) we then obtain 'N ln a' (i.e. ln a^N) on the D scale. If the multiplication was done using the left index of the C scale (see example 1 below), the value of 'a^N' is read off –

(i) The original LL scale for $1 < N < 10$.
(ii) The LL scale above the original for $10 < N < 100$ (e.g. 'a' on the LL_2 then 'a^N' on the LL_3 or 'a' on LL_{02} then 'a^N' on LL_{03}).
(iii) The LL scale below the orginal for $0.1 < N < 1$. (e.g. 'a' on the LL_2 then 'a^N' on the LL_1 or 'a' on LL_{02} then 'a^N' on LL_{01}).

In multiplying 'N' by 'ln a', on the C and D scales, if the right index of the C scale is used, the answers are found on the LL scale above the LL scales listed in the foregoing method. (see example 2).
This method can be extended to larger or smaller number values of N by moving two or more LL scales instead of one.

Example 1: Find $1.4^{2.5}$,1.4^{25}, and $1.4^{0.025}$

1. Set the hair line over 1.4 on the LL_2 scale.
2. Place the left index of the C scale under the hair line.
3. Reset the hair line over 2.5 on the C scale.
   Under the hair line read off –
4. 2.32 on the LL_2 scale as the value for $1.4^{2.5}$.
5. 4,500 on the LL_3 scale as the value for 1.4^{25}.
6. 1.0878 on the LL_1 scale as the value for $1.4^{0.25}$.
7. 1.0084 on the LL_0 scale as the value for $1.4^{0.025}$.

Example 2: Find $0.6^{6.15}$, $0.6^{0.615}$, $0.6^{0.00615}$, $0.6^{-6.15}$, $0.6^{-0.0615}$

1. Set the hair line over 0.6 on the LL_2 scale.
2. Place the right index of the C scale under the hair line.
3. Reset the hair line over 0.615 on the C scale.
   Under the hair line read off –
4. 0.043 on the LL_{03} scale as the value for $0.6^{6.15}$.
5. 0.73 on the LL_{02} scale as the value for $0.6^{0.615}$.
6. 0.99686 on the LL_{00} scale as the value for $0.6^{0.00615}$.
7. 23.3 on the LL_3 scale as the value for $0.6^{-6.15}$.
8. 1.032 on the LL_1 scale as the value for $0.6^{-0.0615}$.

Note:

(a) For negative power we simply transfer to the reciprocal LL scale (see 19.1)
(b) Instead of using the C scale in step 2 and 3 of the above Examples, we could have used the CF scale. Step 2 would then be –Place the index of the CF scale under the hair line. Note, we must use either the C or CF scale in a problem, and not mix them, otherwise it means the power is either multiplied or divided by π.
(c) For powers of numbers which take us outside the range of the LL scales, one of the following procedures can be used.
    (i) $5.9^9 = 5.9^{4.5} \times 5.9^{4.5} = (5.9^{4.5})^2$. Use the LL scales to evaluate $5.9^{4.5}$ and then square the results in the usual way.
    (ii) $36.4^5 = (3.64 \times 10)^5 = 3.64^5 \times 10^5$.
    (iii) $1.52^{29} = 1.52^{19} \times 1.52^{10}$. Evaluate each separately and multiply together in the usual way or $1.52^{29} = 1.52^{14.5} \times 1.52^{14.5} = (1.52^{14.5})^2$
    (iv) $163^{13.8} = (1.63 \times 10^2)^{13.8} = 1.63^{13.8} \times 10^{27.6} = 1.63^{13.8} \times 10^{0.6} \times 10^{27}$. Evaluate the first term using the LL scales. To find the second term, use the L scale as $10^{0.6} = x$ is equivalent to $\log_{10}x = 0.6$ (i.e. find 0.6 on the LL scale and read off 3.98 on the D scale (or W'_2 scale for the 2/83N) as $10^{0.6}$)
    (v) $0.0021^{5.7} = (.21 \times 10^{-2})^{5.7} = 0.21^{5.7} \times 10^{-11.4} = 0.21^{5.7} \times 10^{0.6} \times 10^{-12}$
    Again, we evaluate the first two terms as in (iv) above.

Exercise 19(c)

(i)	$3.7^{6.1} =$	(iii)	$3.7^{-0.61} =$	(v)	$0.179^{4.3} =$
(ii)	$3.7^{0.61} =$	(iv)	$0.625^3 =$	(vi)	$0.02^{1.7} =$

(vii)	$79^2 =$	(xii)	$22^{-1.3} =$	(xvii)	$e^{14} =$
(viii)	$70^{0.14} =$	(xiii)	$44^{-0.75} =$	(xviii)	$1{,}100^{-4.8} =$
(ix)	$79^{-0.002} =$	(xiv)	$1.01^{70} =$	(xix)	$0.0057^{8.3} =$
(x)	$4{,}200^{0.003} =$	(xv)	$0.98^{34} =$	(xx)	$0.042^{-19} =$
(xi)	0.9154.2 =	(xvi)	$14^{22} =$		

19.4 Miscellaneous Powers and Roots of Positive Numbers

Expressing $\sqrt[N]{a} = a^{\frac{1}{N}}$ we can the LL scale to obtain the root of any positive number. As previously with "powers" in 19.3, for 'a' on an LL scale the D scale gives 'ln a'. If we divide this value of 'ln a' by 'N', we then obtain '$\frac{1}{N}\ln a$' (i.e. $a^{\frac{1}{N}} = \ln\sqrt[N]{a}$) on the D scale. We must note the location of the decimal point for '$\frac{1}{N}$' so as to decide on which LL scale the value of '$\sqrt[N]{a}$' will be found. This is done according to the same rules used in 19.3.

Example 1 Find $\sqrt[3]{16}$ and $\sqrt[30]{16}$

1. Set the hair line over 16 on the LL_3 scale.
2. Place the 3 of the C scale under the hair line.
3. Reset the hair line over the right index of the C scale.
   (Note: $\frac{1}{3} \approx 0.33$ and $\frac{1}{30} \approx 0.033$, thus we will find $\sqrt[3]{16}$ on the LL_2 scale and $\sqrt[30]{16}$ on the LL_1 scale.)
   Under the hair line read off –
4. 2.52 on the LL_2 scale as the value for $\sqrt[3]{16}$.
5. 1.0968 on the LL_1 scale as the value for $\sqrt[30]{16}$.

Example 2: Find $\sqrt[5]{420}$ and $\frac{1}{\sqrt[5]{420}}$

1. Set the hair line over 420 on the LL_3 scale.
2. Place the 5 of the C scale under the hair line.
3. Reset the hair line over the left index of the C scale.
   Under the hair line read off –
4. 3.35 on the LL_3 scale as the value for $\sqrt[5]{420}$.
5. 0.298 on the LL_{03} scale as the value for $\frac{1}{\sqrt[5]{420}}$.

Note: If we use the right index of the C scale when we divide by 'N' for 1 < N < 10 the value for '$\sqrt[N]{a}$' will be found on the LL scale below the original LL scale on which 'a' was located. While, if 10 < N < 100 the value for '$\sqrt[N]{a}$' will be found on the second LL scale below the original one.

If we use the left index of the C scale when we divide by 'N', for 1 < N < 10 the value for '$\sqrt[N]{a}$' will be located on the same LL scale on which 'a' was located. While, if 10 < N < 100 the value for '$\sqrt[N]{a}$' will be found on the LL scale below the original one.
Some special powers are given in the following table.

On appropriate LL scale Set the H.L over -	Under the H.L place	Reset H.L. over	On appropriate LL scale under H.L answer

a	Index of CI scale	N on the CI scale	$a^{\frac{1}{n}}$
a	Index B	N B	$a^{\sqrt{N}}$
a	Index BI	N BI	$a^{\frac{1}{\sqrt{N}}}$
a	Index K’	N K’	$a^{\sqrt[3]{N}}$
a	Index W	N W	a^{N^2}
a	Index C	N CF	$a^{\frac{N}{\pi}}$
a	Index CF	N C	$a^{n\pi}$
a	N CI	M C	a^{NM}
a	M C	N C	$a^{\frac{N}{M}}$
a	N C	M CI	$a^{\frac{1}{NM}}$
a	Index C	θ S’	$a^{\sin\theta}$

Exercise 19(d)

(i) $\sqrt[4]{280} =$

(ii) $\sqrt[7]{1.85} =$

(iii) $\sqrt[9]{0.64} =$

(iv) $\sqrt[3]{7.8} =$

(v) $\sqrt[14]{0.78} =$

(vi) $\frac{1}{\sqrt[5]{33}} =$

(vii) $3.2^{\frac{1}{5.3}} =$

(viii) $1.26^{\sqrt{13}} =$

(ix) $52^{\sin 53} =$

(x) $0.75^{\frac{1}{\sqrt{0.9}}} =$

(xi) $4^{\sqrt[3]{30}} =$

(xii) $0.96^{4.1^2} =$

(xiii) $1.05^{3\pi} =$

(xiv) $121^{\frac{1}{6\times 2.4}} =$

(xv) $2{,}100^{\frac{0.1}{13}} =$

(xvi) $0.7^{3.6\times 4.1} =$

19.5 Logarithms To Any Base and Solving Exponential Equaltions

A. Logarithm

The LL scales can be used to obtain logarithms to any base, by placing the left or right index of the C scale over the base as found on the LL scale. Then for any number on an LL scale its logarithm to the chosen base is read off the C scale.

Example 1: $\text{Log}_6 23.5=1.765$

1. Set the hair line over 6 on the LL_3 scale.
2. Place the left index of the C scale under the hair line.
3. Reset the hair line over 23.5 on the LL_3 scale.
4. Under the hair line read off 1.765 on the C scale as the answer.

Note:

(a) We could have used the CF scale above, by placing the index of the CF scale under the hair line in step 2, and thus reading 1.765 off the CF scale in step 4.

(b) For logarithms of other numbers to base 6, leave the slide as positioned in step 2 and reset the hair line over the number on an LL scale. (e.g. $\log_6 2 = 0.3875$ as 2 on the LL_2 scale give 0.3875 on the C scale.)

Example 2: Find $\log_2 1.4$, $\text{Log}_2 23$ and $\log_2 650$.

1. Set the hair line over 2 on the LL_2 scale.
2. Place the right index of the C scale under the hair line.
3. Reset the hair line over 1.4 on the LL_2 scale and read off 0.485 on the C scale as the value for $\log_2 1.4$.

4. Reset the hair line over 23 on the LL_3 scale and read off 4.52 on the C scale as the value for $\log_2 23$.
5. Reset the hair line over 650 on the LL_3 scale and read off 9.34 on the C scale as the value for $\log_2 650$.

B. Solving exponential equations

Because $\text{Log}_b N = x$ is equivalent to $N = b^x$ the above problems are the same as solving an exponential equation for an unknown power. Example 1 could have been stated – Find x for 6x = 23.5. Thus to solve for an unknown power or exponent we could either express the equation in logarithmic form or leaving it as an exponential equation proceed as follows for $N = b^x$.

1. Set the hair line over the base 'b' on the appropriate LL scale.
2. Place the left or right index of the C scale under the hair line.
3. Reset the hair line over the number 'N' on its appropriate LL scale.
4. Under the hair line read off the value for x on the C scale and locate the decimal point according to the LL scales used.

Note:

(a) To solve the equation $b^{\frac{1}{x}} = N$ for x, we follow the same method as outlined above, except in step 4 the value of x is read off the CI scale.

(b) To solve the equation $b^{kx} = N$ for x, the following method can be used –

Example: $e^{1.4x} = 9$

1. Set the hair line over e on the LL_3 scale.
2. Place the 1.4 of the CIF scale under the hair line. (We can also use CI scale here if suitable).
3. Reset the hair line over 9 on the LL_3 scale.
4. Under the hair line read off 1.57 on the CF scale as the value for x.
   (When the CI scale is used in 2, the answer in 4 will be on the C scale.)

Exercise 19(e)

(i) $\log_5 1.9 =$

(ii) $\log_8 24 =$

(iii) $\log_2 57 =$

(iv) $\log_3 0.53 =$

(v) $\log_{1.5} 2 =$

(vi) $\log_{15} 1.3 =$

(vii) $\log_5 260 =$

(viii) $\log_{2.5} 17 =$

Find x in the Following:

(ix) $5^x = 30$

(x) $12^x = 76$

(xi) $15^x = 3.5$

(xii) $456^x = 21$

(xiii) $0.3^x = 0.95$

(xiv) $0.16^x = 0.045$

(xv) $2.5^{\frac{1}{x}} = 1.02$

(xvi) $16e^x = 48$

(xvii) $e^{\pi x} = 64$

(xviii) $e^{-3x} = 0.45$